EE-557
Semiconductor devices I

Continuity and Shockley equations

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Master Program
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Outline of the lecture

- Continuity equations
- Shockley equations
- Examples and exercises

Read Chapter 5 (from Shockley equations) of the reference book (on moodle)

References:

- J. A. del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/)

So far we have learned:

- Recombination
- Generation
- Drift
- Diffusion

How to describe carrier flow in semiconductors taking into account all these mechanisms?

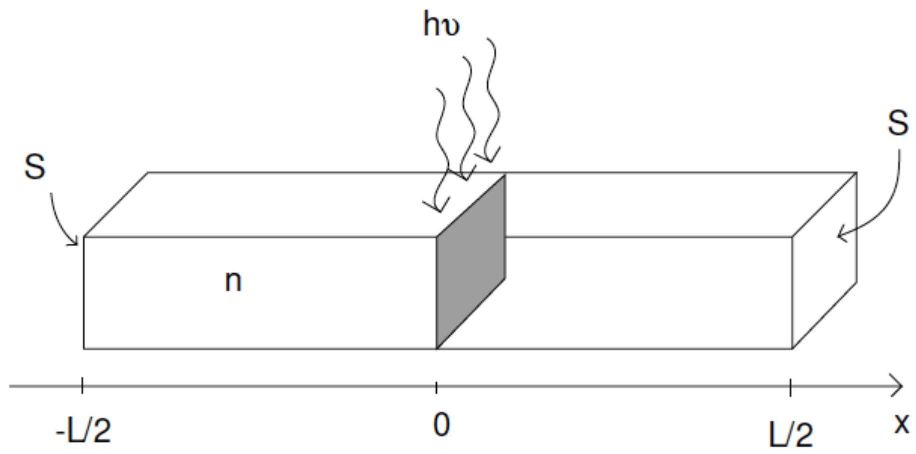
Carrier flow

Carrier dynamics:

$$\frac{dn}{dt} = \frac{dp}{dt} = G - R$$

Applies only to uniform problems in space!

This doesn't work in problems like this: Generation of carriers in a given position of space



Equation system does not capture:

- **Impact of carrier movement** on carrier concentration
  (i.e. when carriers move away from a point, their concentration drops!)

- **Boundary conditions** (surfaces are not infinitely far away – covered in Del Alamo-Chapter 5)

We need to include the flow of carriers

Continuity equations:

For electrons:
$$\frac{\partial n}{\partial t} = G - R + \frac{1}{q}\vec{\nabla}.\vec{J}_e$$

For holes:
$$\frac{\partial p}{\partial t} = G - R - \frac{1}{q}\vec{\nabla}.\vec{J}_h$$

Semiconductor physics so far: Shockley's equations

Gauss' law: $\vec{\nabla} \cdot \vec{\mathcal{E}} = \frac{q}{\epsilon}(p - n + N_D^+ - N_A^-)$

Electron current equation: $\vec{J}_e = -qn\vec{v}_e^{drift} + qD_e\vec{\nabla}n$

Hole current equation: $\vec{J}_h = qp\vec{v}_h^{drift} - qD_h\vec{\nabla}p$

Electron continuity equation: $\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\vec{\nabla} \cdot \vec{J}_e$

Hole continuity equation: $\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\vec{\nabla} \cdot \vec{J}_h$

Total current equation: $\vec{J}_t = \vec{J}_e + \vec{J}_h$

System of non-linear, coupled partial differential equations: generally not solvable in a closed form.

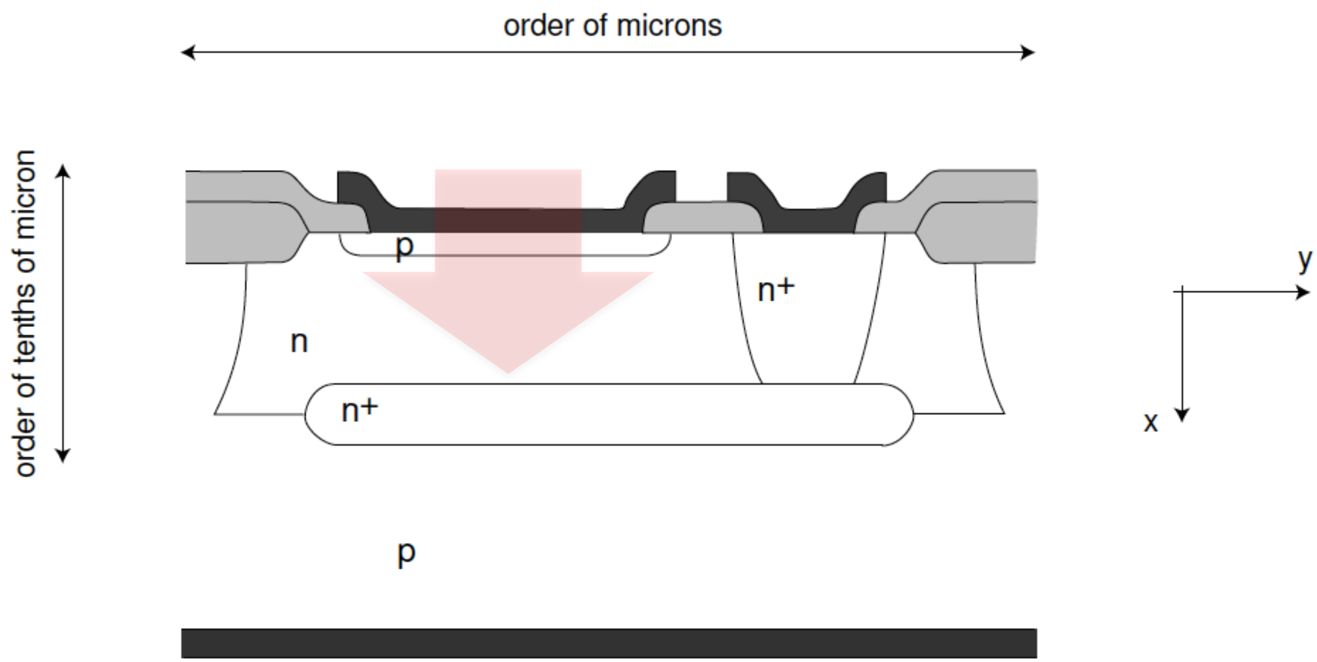
*** Attention: Here the net recombination rate *U(n,p)* includes all the absolute net recombination rates, including surface recombination, which were not discussed in this course.

One-dimensional approximation

1D approximation: $\vec{\nabla} \Rightarrow \frac{\partial}{\partial x}$

In many cases, complex problems can be broken into several 1D sub-problems.

Example: integrated p-n diode



Shockley equations in 1D quasi-neutral situations

in 1D:

Gauss' law: $\frac{\partial \mathcal{E}}{\partial x} = \frac{q}{\epsilon}(p - n + N_D - N_A)$

Electron current equation: $J_e = -qnv_e^{drift}(\mathcal{E}) + qD_e\frac{\partial n}{\partial x}$

Hole current equation: $J_h = qpv_h^{drift}(\mathcal{E}) - qD_h\frac{\partial p}{\partial x}$

Electron continuity equation: $\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\frac{\partial J_e}{\partial x}$

Hole continuity equation: $\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\frac{\partial J_h}{\partial x}$

Total current equation: $J_t = J_e + J_h$

Equation set difficult because of coupling through Gauss' law

Two broad classes of important situations to break Gauss' law coupling:

1. Carrier concentrations are high: **quasi-neutral situation**
   - majority carrier concentration nearly tracks the doping concentration
   - Any introduction of extra carriers in a volume is negligible compared to the charge density already present

$$\rho \simeq 0 \Rightarrow \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

2. Carrier concentrations are very low: **space-charge** and **high-resistivity situations**: **E independent of n, p**

Quasi-neutral (QN) approximation
At every location, the net volume charge that arises from a discrepancy of the concentration of positive and negative species is **negligible** in the scale of the charge density that is present.

$$\rho \simeq 0 \;\Rightarrow\; \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

QN approximation eliminates Gauss' law from the set:

$$\rho = q(p - n + N_D^+ - N_A^-) = q(p_o - n_o + N_D^+ - N_A^-) + q(p' - n')$$

Quasi-neutrality out of equilibrium:

$$\left|\frac{p' - n'}{n'}\right| \simeq \left|\frac{p' - n'}{p'}\right| \ll 1$$

$$p' \simeq n'$$

QN approximation is good if **n, p high** ⇒carriers move to erase ρ.

Simplifications of Shockley equations to 1D quasi-neutral situations

Consequence of quasi-neutrality

$$\rho \simeq 0 \;\Rightarrow\; \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

$$\mathcal{E} = \mathcal{E}_o + \mathcal{E}'$$

Then, in equilibrium:

$$\frac{\partial \mathcal{E}_o}{\partial x} = \frac{q}{\epsilon}(p_o - n_o + N_D^+ - N_A^-)$$

and out of equilibrium:

$$\frac{\partial \mathcal{E}'}{\partial x} = \frac{q}{\epsilon}(p' - n')$$

Simplifications of Shockley equations to 1D quasi-neutral situations

Consequence of quasi-neutrality

Subtract one continuity equation from the other:

(1) $\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\frac{\partial J_e}{\partial x}$

(2) $\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\frac{\partial J_h}{\partial x}$

(1) – (2): $$\frac{\partial J_t}{\partial x} = q\frac{\partial (n-p)}{\partial t} = -\frac{\partial \rho}{\partial t}$$

Total current must be continuous everywhere: **current continuity**

Continuity equation for net volume charge: **if J_t changes with position, ρ changes with time**.

- In *Static case*:

$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial J_t}{\partial x} = 0,\ J_t \text{ independent of } x$$

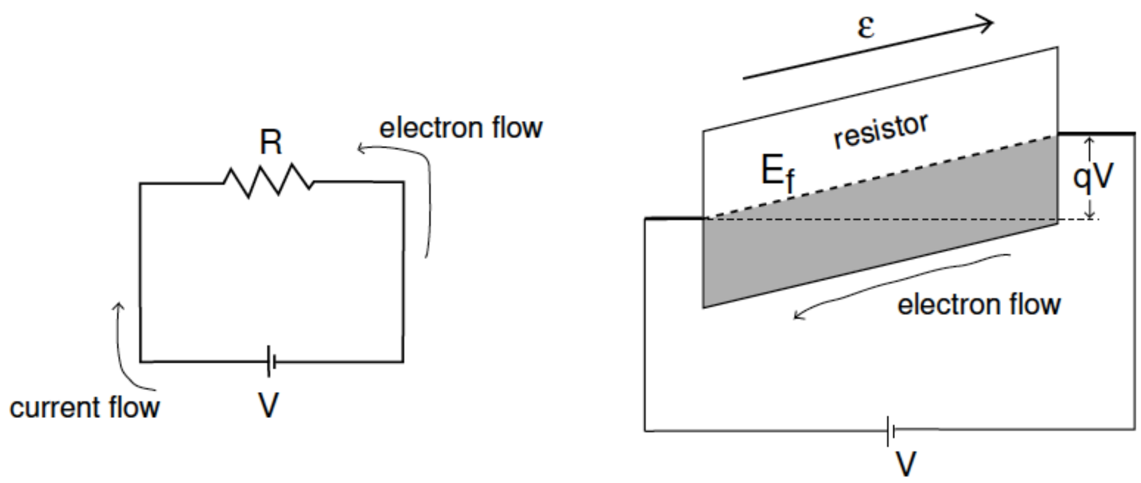
- In *Dynamic case*, we also have in most useful situations:

$$\frac{\partial \rho}{\partial t} \simeq 0 \text{ in times scale of interest}$$

Case 1: Majority-carrier type situations

Semiconductor resistor

Voltage applied to extrinsic quasi-neutral semiconductor, without changing the equilibrium carrier concentrations:



circuit view **energy view**

The battery picks up electrons from positive terminal, increases their potential energy and puts them at the negative terminal.

Energy given to electrons: **E = qV**

If provided with a path (resistor), electrons flow.

Case 1: Majority-carrier type situations

Semiconductor resistor

Characteristics of majority carrier-type situations:

- Small electric field imposed from outside: does not disturb the dynamic balance existing in equilibrium
- Carriers are not disturbed from equilibrium: time derivatives are zero!
- electrons and holes drift $\frac{dJ_e}{dx} \simeq 0, \frac{dJ_h}{dx} \simeq 0, \frac{dJ_t}{dx} \simeq 0$

Electron and hole concentrations unperturbed from TE Simplifications:

- neglect contribution of minority carriers
- neglect time derivatives of carrier concentrations

⇒ problem becomes completely quasi-static
Equation set for 1D majority-carrier type situations:

<table>
<tr><th>n-type</th><th>p-type</th></tr>
<tr><td>$n \simeq n_o \simeq N_D$</td><td>$p \simeq p_o \simeq N_A$</td></tr>
<tr><td>$J_e = -qn_o[v_{de}(\mathcal{E}) - v_{de}(\mathcal{E}_o)]$</td><td>$J_h = qp_o[v_{dh}(\mathcal{E}) - v_{dh}(\mathcal{E}_o)]$</td></tr>
<tr><td colspan="2">$\frac{dJ_e}{dx} \simeq 0, \frac{dJ_h}{dx} \simeq 0, \frac{dJ_t}{dx} \simeq 0$</td></tr>
<tr><td>$J_t \simeq J_e$</td><td>$J_t \simeq J_h$</td></tr>
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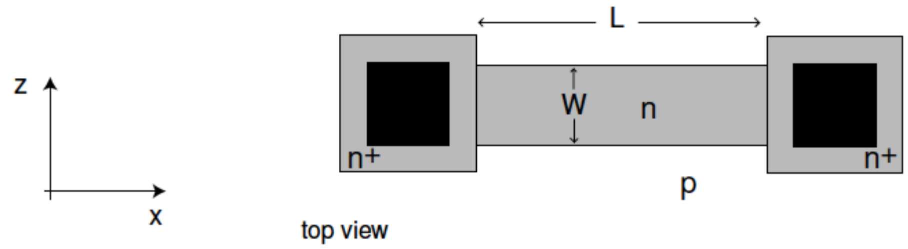
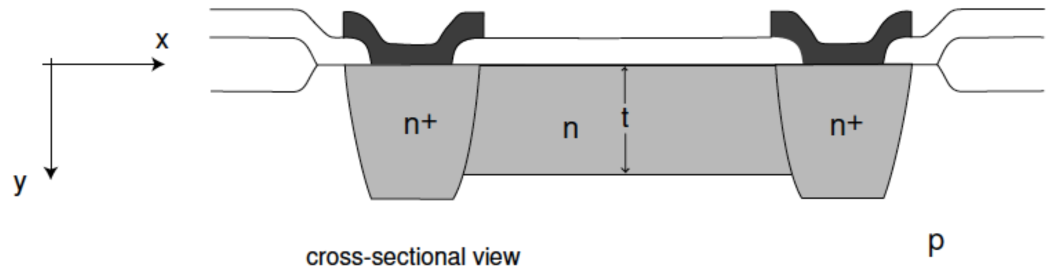
Simplifications of Shockley equations to 1D quasi-neutral situations

Integrated Resistor with uniform doping (n-type)





Equations:

$$J_t = -qN_D v_e^{drift}(\mathcal{E})$$

$$J_t \simeq qN_D \mu_e \mathcal{E}$$

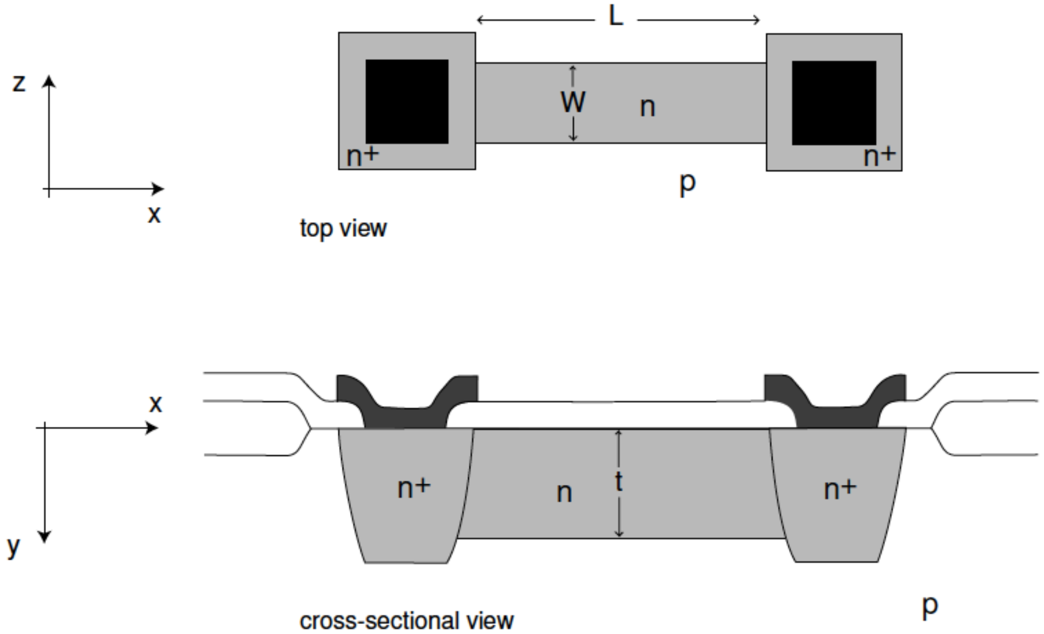
$$I = WtqN_D\mu_e \frac{V}{L}$$

In general (low and high fields):

$$I = WtqN_D \frac{v_{sat}}{1 + \frac{v_{sat}}{\mu_e}\frac{L}{V}}$$

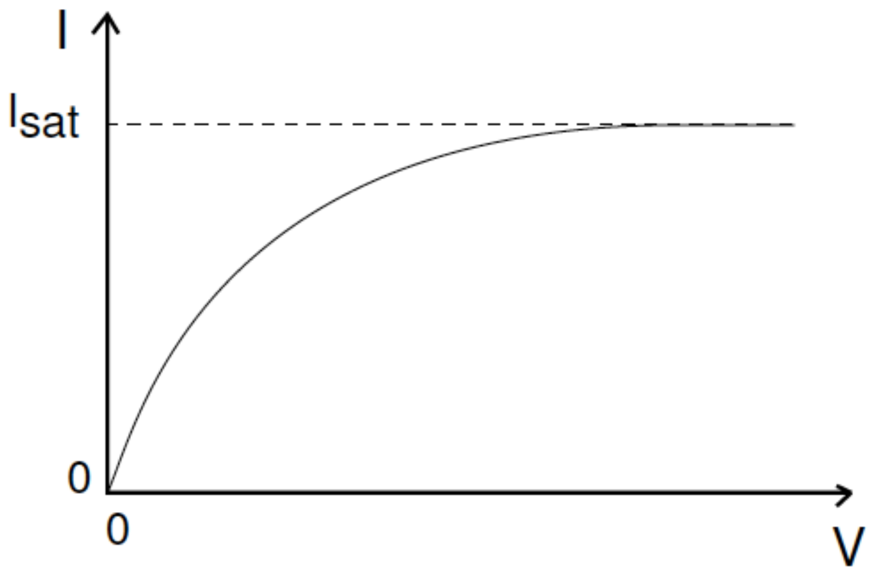
Simplifications of Shockley equations to 1D quasi-neutral situations

Integrated Resistor with uniform doping (n-type)



which for high fields saturates to:

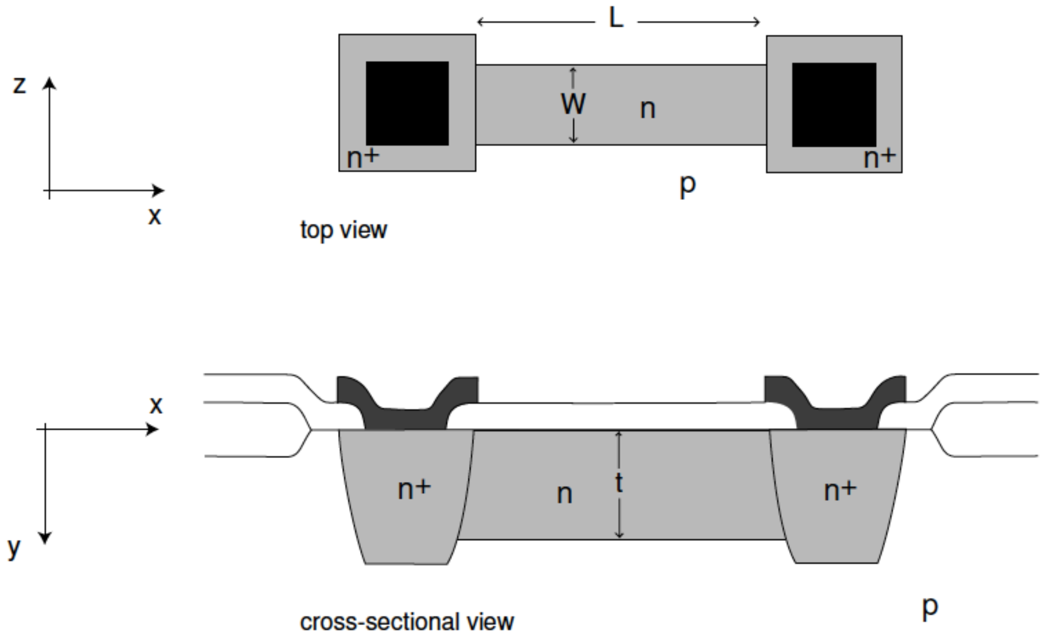
$$I_{sat} = WtqN_D v_{sat}$$

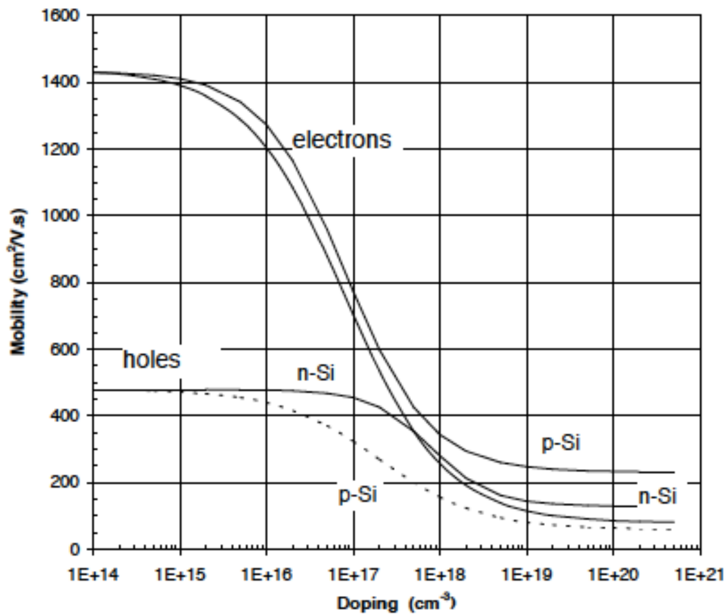


Exercise:

Consider an integrated resistor composed of an n-type Si layer fabricated on a p-type Si. The doping level is $N_D = 10^{19} cm^{-3}$. The dimensions of its active region are L=5um W=2um and t=0.1um. At RT, estimate:

1. Sheet resistance of the n-type semiconductor
2. Resistance of the resistor.





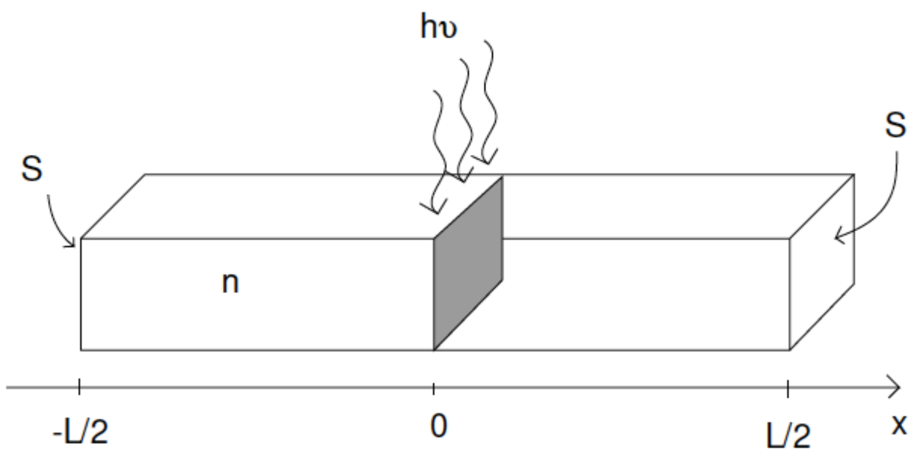
Case 2: Minority-carrier type situations

- Low level injection
- Excess carriers
- No significant electric field
- Quasi-neutrality

n-type	p-type
$p_o - n_o + N_D - N_A \simeq 0$	
$p' \simeq n'$	
$J_e = qn_o\mu_e\mathcal{E}' + qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$	$J_e = qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$
$J_h = qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$	$J_h = qp_o\mu_h\mathcal{E}' + qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$
$D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t}$	$D_e\frac{\partial^2 n'}{\partial x^2} + \mu_e\mathcal{E}_o\frac{\partial n'}{\partial x} - \frac{n'}{\tau} + G_{ext} = \frac{\partial n'}{\partial t}$
$\frac{\partial J_t}{\partial x} \simeq 0$	
$J_t = J_e + J_h$	

Case 2: Minority-carrier type situations

- Let's look at a n-type bar



Shockley equations:
system of equations that describes carrier phenomena in semiconductors in the drift-diffusion regime.

Quasi-neutral approximation:
appropriate if semiconductor is sufficiently extrinsic: ρ ~ 0 ⇒

$$n_o - p_o \simeq N_D - N_A \qquad n' \simeq p'$$

Majority carrier-type situations characterized by application of **external voltage without perturbing carrier concentrations**.

Majority-carrier type situations dominated by drift of majority carriers.
Integrated resistor:

- for low voltages, current proportional to voltage across
- for high voltages, current saturates due to v^{sat}